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BRIEF COMMUNICATION

INERTIA AND SURFACE TENSION EFFECTS IN NEWTONIAN LIQUID JETS

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INTRODUCTION

The analysis of the laminar flow of an incompressible Newtonian liquid jet issuing from a circular nozzle into an inviscid gas phase has received considerable attention in recent years. Harmon (1955), Slattery & Schowalter (1964) and Joseph (1974) utilized various forms of the macroscopic momentum balance to derive expressions for the final diameter of a horizontal capillary jet. Duda & Vrentas (1967) obtained finite-difference solutions for both vertical and horizontal jets at the high Reynolds number limit where the boundary layer assumptions are valid and the governing differential equations are parabolic. Surface tension effects were included in this analysis. These investigators also obtained an analytical solution of a linearized form of the equations of motion. Horsfall (1973), Nickell *et al.* (1974), Reddy & Tanner (1978) and Omodei (1980) obtained numerical solutions of the elliptic partial differential equations describing jet flow for a wide Reynolds number range with and without surface tension. Middleman & Gavis (1961), Goren & Wronski (1966), Gavis & Modan (1967) and Bilgen (1971) experimentally studied the shape of Newtonian liquid jets issuing from a capillary into air. Finally, techniques used in the analysis of liquid-gas jets have recently been applied in theoretical studies of liquid–liquid laminar jets (Yu & Scheele 1975, Gospodinov *et al.* 1979). The purpose of this brief communication is to utilize the results of the above studies to comment on some aspects of inertia and surface tension effects in a horizontal Newtonian liquid jet flowing into a continuous gas phase.

THEORETICAL CONSIDERATIONS

It can be easily shown that the radius, R, of a horizontal jet of an incompressible Newtonian liquid is represented by an equation of the following functional form

$$\frac{R}{R_0} = F_1\left(\frac{z}{R_0}, \mathrm{We}, \mathrm{Re}\right) \quad [1]$$

where R_0 is the nozzle radius and z is the axial distance from the nozzle. The Reynolds number, Re, and the Weber number, We, are defined as follows

$$\mathrm{Re} = \frac{2R_0\rho U_a}{\mu} \quad [2]$$

$$\mathrm{We} = \frac{2R_0U_a^2\rho}{\sigma} \quad [3]$$

where ρ, μ, and σ are the density, viscosity, and surface tension of the liquid and U_a is the average velocity in the jet nozzle. Below, we examine just how strongly the surface tension influences the jet shape as the Reynolds number is varied from the creeping flow asymptote to the boundary layer limit.

For sufficiently high Reynolds numbers (but where the flow is still laminar), the equations of motion assume their boundary layer form. Both the radial pressure gradient and axial diffusion of vorticity are negligible, and the jet shape is described by an equation of the following form:

$$\frac{R}{R_0} = F_2\left(\frac{z}{\mathrm{Re}\, R_0}, \mathrm{We}\right). \qquad [4]$$

In addition, the macroscopic momentum balance yields the following expression for the final diameter of a horizontal jet at the high Reynolds number limit (Slattery & Schowalter 1964, Duda & Vrentas 1967)

$$0 = \frac{2}{\mathrm{We}}(\alpha^3 - \alpha^2) + \frac{4}{3}\alpha^2 - 1 \qquad [5]$$

where

$$\alpha = \frac{R(z = \infty)}{R_0}. \qquad [6]$$

In the limit of negligible surface tension ($\mathrm{We} \to \infty$), we recover the well known result, $\alpha = \sqrt{3}/2$, from [5]. We now examine how rapidly the boundary layer flow is approached as the Reynolds number is increased above the creeping flow limit.

DISCUSSION

The numerical results of Omodei (1980) for the dependence of jet radius on jet length in the absence of surface tension are presented in figure 1 for three Reynolds numbers. It is evident from this figure that a Reynolds number greater than 1000 must be reached before the boundary layer limit is effectively achieved. The boundary layer result shown in figure 1 is of the form of [4] and is based on the theoretical analysis of Duda & Vrentas (1967). Experimental data for the final jet radius from three independent studies are presented as a function of Reynolds number in figure 2. The curves in this figure are average representations of the data points of each of the three investigations. The data of Bilgen (1971) were taken under conditions where interfacial effects are important. A possible explanation for the reduction in the jet contraction which was observed by Bilgen and which is evident from figure 2 is given below. Although the data of Middleman & Gavis (1961) and of Gavis & Modan (1967) are relatively free of surface tension effects, the agreement between the data is not particularly good. The scatter in the data of Middleman & Gavis is significantly greater than that in the data of Gavis & Modan. If we thus assume that the latter data set is more reliable, then the experimental data appear to support the previously stated conclusion: A Reynolds number greater than 1000 is needed to reach the boundary layer asymptote.

The jet flow can be regarded as a stick-slip flow field (Trogdon & Joseph 1981). The flow field changes from a fully developed flow in a pipe with a no slip condition on the wall to a uniform flow field far downstream with a no drag condition. Hence, the fluid flows from a high vorticity region into one where the vorticity is very low.

The conclusion that the boundary layer limit is reached only at relatively high Reynolds numbers for the stick-slip jet flow is dramatically different from the result for a slip-stick flow. In the latter flow field, fluid flows from an impermeable, frictionless tube of infinite extent (where the velocity profile is initially uniform) into an infinite pipe with a no slip condition on the wall. For this case, the flow of fluid is from a low vorticity region to a high vorticity region

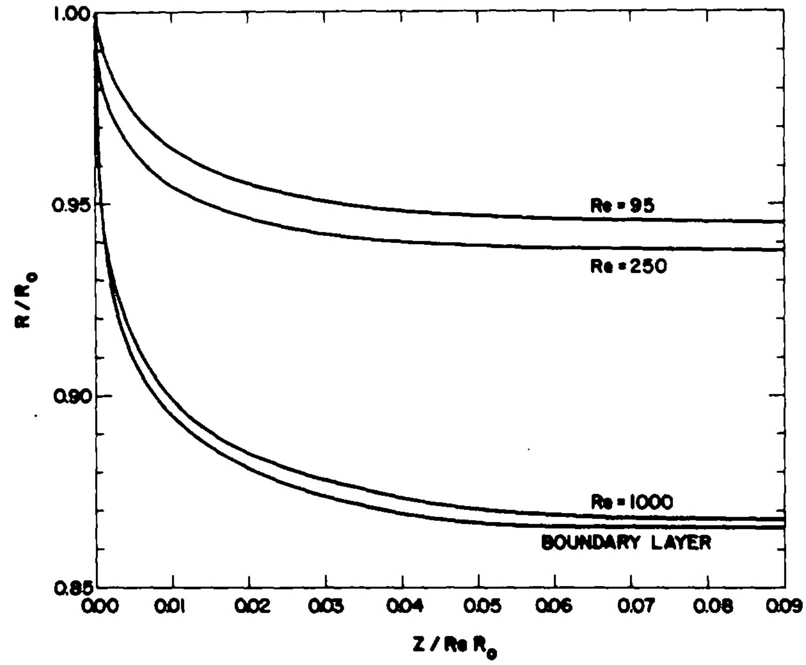

Figure 1. Dependence of jet radius on jet length in the absence of surface tension. Curves are based on numerical solutions of Omodei (1980) and boundary layer theory of Duda & Vrentas (1967).

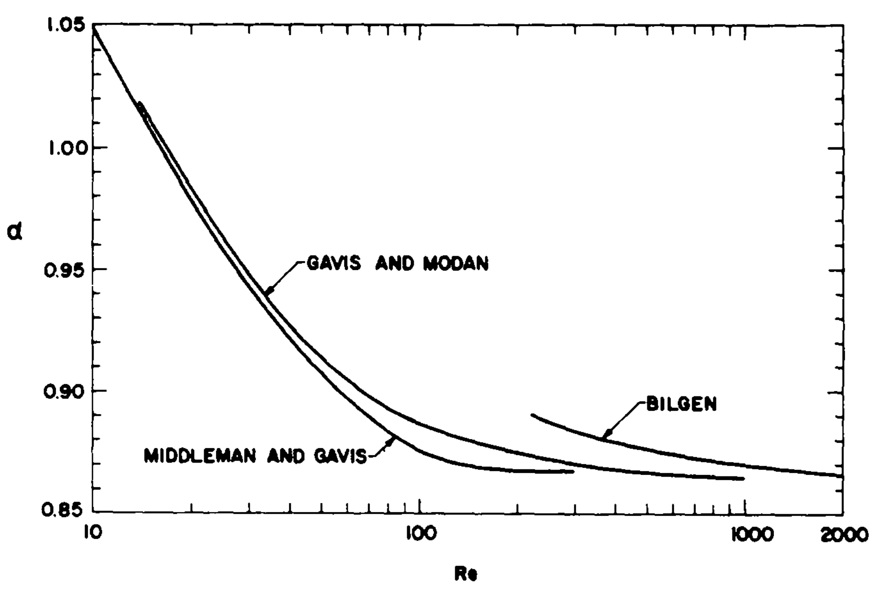

Figure 2. Experimental data for final jet radius of a horizontal jet.

since vorticity is continuously being generated at the solid surface of the tube. This slip-stick flow represents an early model of laminar entrance flow into a pipe which was studied by Vrentas *et al.* (1966) and by Vrentas & Duda (1967). The dimensionless axial velocity at the tube center line for the slip-stick flow is plotted versus axial position in figure 3. For this flow field, we see that the boundary layer asymptote is achieved in the vicinity of Re = 150.

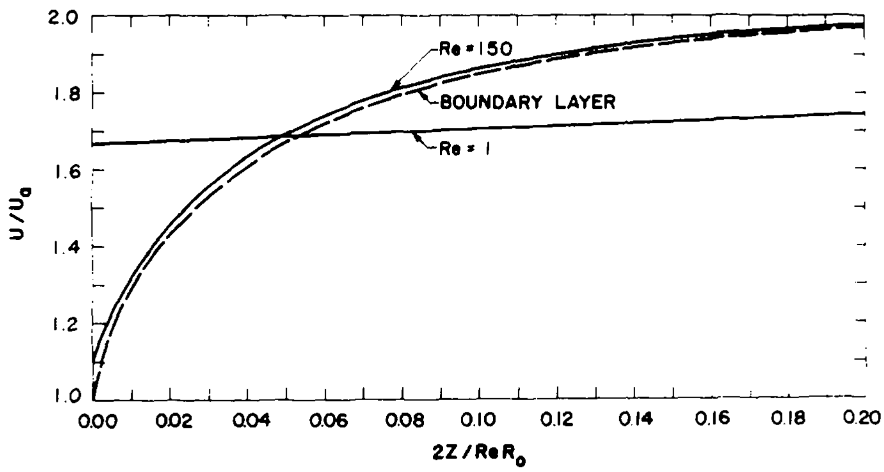

Figure 3. Dimensionless axial velocity at center line of tube versus axial position from tube entrance for slip-stick flow.

The difference between the stick-slip jet exit flow and the slip-stick entrance flow model can be explained by considering the relative strengths of the axial diffusion and the axial convection of vorticity. Axial diffusion of vorticity is dominant in the creeping flow limit where the Reynolds number is low and the convective transport of vorticity is weak. As the Reynolds number is increased, the convective motion begins to overwhelm the diffusive transfer. Ultimately, the effect of axial diffusion of vorticity becomes negligible and the boundary layer limit is reached. For the jet flow, convective flow of high vorticity fluid near the tube wall is not particularly effective because of the low velocities in that region. In contrast, for the entrance flow model where the velocity profile is initially uniform, there exists a strong convective flow of low vorticity fluid everywhere in the frictionless stream tube. Consequently, it is reasonable to expect that significantly higher Reynolds numbers are needed for the jet flow to wash out the effect of the axial diffusion of vorticity due to the existence of relatively weak convective flows for this configuration.

The effect of surface tension on the final jet radius is presented in figure 4 for various Reynolds numbers. In this figure, the value of α in the absence of surface tension, α (We $= \infty$), is compared to the value of α when surface tension is operative, α (We). The boundary layer curve was computed from [5], and the curves for intermediate Reynolds numbers were derived from the results of Omodei (1980). For the creeping flow limit (Re $= 0$), it can be shown that there is no surface tension effect for We > 0. It is evident from this figure that, for most of the Reynolds number range, the effect of surface tension can be significant for sufficiently low We. At a particular value of We, the most significant effects occur at high Reynolds numbers and at a Reynolds number relatively close to the creeping flow asymptote. This is shown in figure 5 where, for We $= 4$, the greatest effects of surface tension are at the boundary layer limit and at a Reynolds number between 0 and 5. Dashed lines in figure 5 denote possible interpolations.

As noted above, surface tension can exert a very significant influence on jet shape for both high and low Reynolds numbers if the Weber number is sufficiently low (say, less than 10). Low velocities lead to low values of both Re and We. It therefore follows, as noted previously by Joseph (1974), that surface tension has a large effect on low speed, and hence low Reynolds number, jets. However, it is not likely that this large effect will be observed in creeping flow jets in the laboratory since it may be difficult to approach this limit by lowering the jet velocity. A liquid jet is almost certainly unstable at low speeds, and the low Reynolds number region is usually approached experimentally by using fluids of higher viscosity (Middleman & Gavis

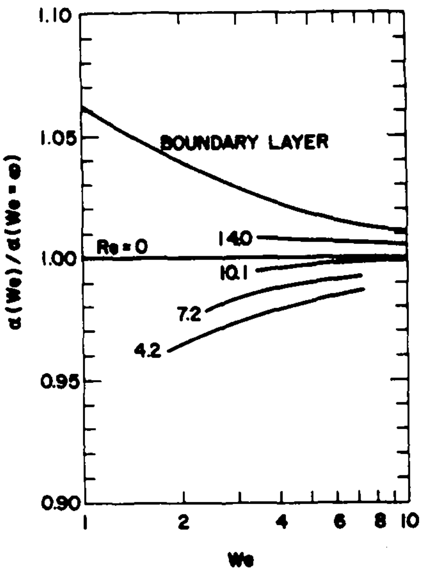

Figure 4. Effect of surface tension on final jet radius for various values of Re. Boundary layer curve was computed from [5] and results for intermediate Reynolds numbers are from Omodei (1980).

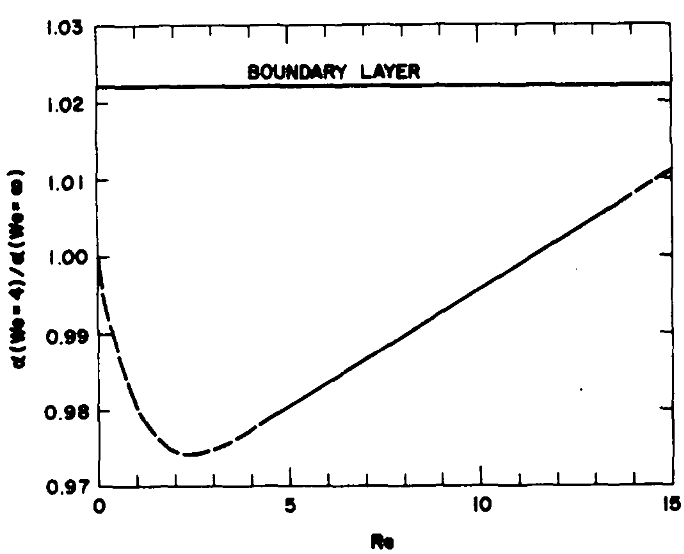

Figure 5. Effect of surface tension on final jet radius for We = 4 and various values of Re. Dashed lines indicate possible interpolations.

1961, Goren & Wronski 1966, Gavis & Modan 1967). Consequently, in practice, the Weber number for a low Reynolds number jet is not necessarily particularly small, and the influence of surface tension on that jet shape could very well be negligible in the creeping flow region. We note that, in all three studies cited above, most of the data were taken with We > 10 even though Reynolds numbers less than five were achieved.

Finally, we remark that it is possible to experimentally observe significant surface tension effects at moderately high Reynolds numbers. This is contrary to the statement made by Omodei (1980) that the effect of surface tension is negligible for Re > 65 for experimentally realizable jets. Bilgen (1971) presented data for a jet with $Re \approx 200$ and $We \approx 2$. We see from figure 4 that a very significant influence of surface tension on the final diameter is predicted for this case. Since surface tension produces thicker jets for $Re > 11$, it is reasonable to conclude that at least part of the reason that the jets observed by Bilgen were larger than those observed by Gavis & Modan (1967) is the presence of surface tension effects. We thus conclude that significant surface tension effects on the jet shape are not necessarily limited to low Reynolds numbers. This conclusion does not question the validity of the numerical results of Omodei but rather his conclusion that significant surface tension effects cannot be observed in the laboratory at high Reynolds numbers.

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